PHYSICS 3900F/G

COMPTON SCATTERING OF GAMMA RAYS

1. Introduction

When a photon of energy E_0 interacts with a free electron of mass m and charge e, the photon may be scattered. If the photon has an initial momentum p_0, its momentum p_f after scattering will make some angle θ with respect to p_0. In order to conserve both energy and momentum in the scattering, the photon must give up some energy in the scattering, and the electron will recoil with some kinetic energy at some angle ϕ.

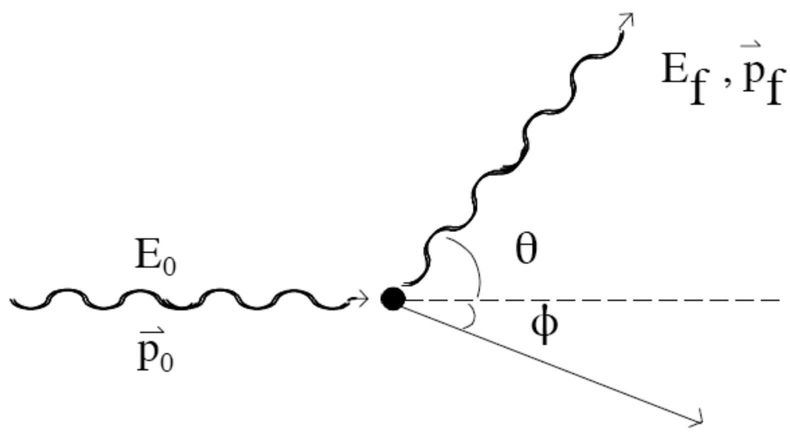

Fig.1. Compton Scattering Process

You should show using standard relativistic conservation laws that the final energy E_f of a gamma ray scattered through angle θ is given by

$$E_f = \frac{E_0}{1 + \alpha(1 - \cos\theta)} \tag{1}$$

where $\alpha = E_0/mc^2$. Why is it a good approximation to consider the electron to be free and at rest in the laboratory frame?

In an experimental measurement, a scatterer of thickness t, containing n_e electrons per unit volume, is irradiated by a beam of γ-rays so that N photons per second are incident on the scatterer. The scattered γ-rays are observed by a γ-detector at some angle with respect to the incident beam. The solid angle subtended at the target by an element of the detector at angle θ is $d\Omega$. Each electron has an effective area $d\sigma$ for scattering a γ-rays into this solid angle $d\Omega$, which can be written $d\sigma = (d\sigma\ /d\Omega)d\Omega$, where $d\sigma/d\Omega$ is defined as the differential cross section for

Compton scattering at angle θ. If A is the cross section of the beam intersecting the scatterer, then the number of photons per second scattered into the solid angle $d\Omega$ by a single electron in the scatterer will be $(N/A)d\sigma$; the number of electrons in the scatterer is $An_e t$, so the number of photons per second arriving at the detector element will be

$$(N/A)\,d\sigma\left(An_e t\right) = Nn_e t\left(d\sigma/d\Omega\right)d\Omega$$

If the efficiency of the detector is ε, then the counting rate R in the detector will be

$$R = \varepsilon N n_e t \int \left(d\sigma/d\Omega\right)d\Omega$$

integrated over the entire detector solid angle $\Delta\Omega$. The counting rate R will vary with γ-ray energy through the dependence on energy of ε and $d\sigma/d\Omega$.

The differential cross section as a function of θ can be calculated by classical physics if the γ-ray energy is low, i.e. $\alpha << 1$. (Note however that E_0 cannot be so low that the free-electron approximation is invalidated.). This is called the Thompson cross section:

$$\left(\frac{d\sigma}{d\Omega}\right)_{\text{thomson}} = \frac{1}{2r_0^2\left(1+\cos^2\theta\right)} \tag{2}$$

where $r_0 = e^2/(4\pi\varepsilon_0 mc^2)$ is known as the *classical radius of the electron.*

For high-energy gamma rays, the non-relativistic approximation breaks down and the differential cross section is given by the Klein-Nishina formula:

$$\left(\frac{d\sigma}{d\Omega}\right)_{\text{k-n}} = \left(\frac{d\sigma}{d\Omega}\right)_{\text{thomson}}\left[\frac{1}{1+\alpha(1-\cos\theta)}\right]^3\left[1+\frac{\alpha^2(1-\cos\theta)^2}{\left(1+\cos^2\theta\right)\left(1+\alpha(1-\cos\theta)\right)}\right] \tag{3}$$

In this experiment you will be able to observe the Compton scattering of γ-rays from a ^{137}Cs source (E_0= 0.661 MeV). You will be able to verify the prediction of Eq. (1) with reasonable accuracy, and you should be able to obtain qualitative verification of Eq. (3).

Before starting this experiment you should carry out the following:

1. Plot $E_f(\theta)$ from Eq. (1) for ^{137}Cs γ-rays.
2. Plot $(d\sigma/d\Omega)_{\text{k-n}}$ vs θ from Eq. (3).
3. Calculate the electron density for lucite, aluminum and copper, and compare the expected relative counting rates at a given angle for scatterers of the same physical thickness.
4. Using Fig. 3 (available in the lab), find the efficiency ε as a function of γ-ray energy for the detector used in the measurement. Note that what is actually

plotted in Fig. 3 is the total absorption coefficient, γ, due to all processes. Thus, after passing through a thickness d of the crystal, the fraction of the incident gamma-ray energy absorbed in the crystal is $1 - \exp(-\gamma d)$.

5. Familiarize yourself with the radioactive hazards associated with the experiment, and the procedures to be followed in using γ-ray sources. As a minimum you must be familiar with the following pages from the UWO Radiation Safety Manual [3] available in the lab: pp.13, 16, 48–63. (You may skip the material on beta radiation if you wish.)

2. Procedure

1. Spend time familiarizing yourself with the apparatus and layout of the experiment. A schematic diagram is shown in Fig. 2.

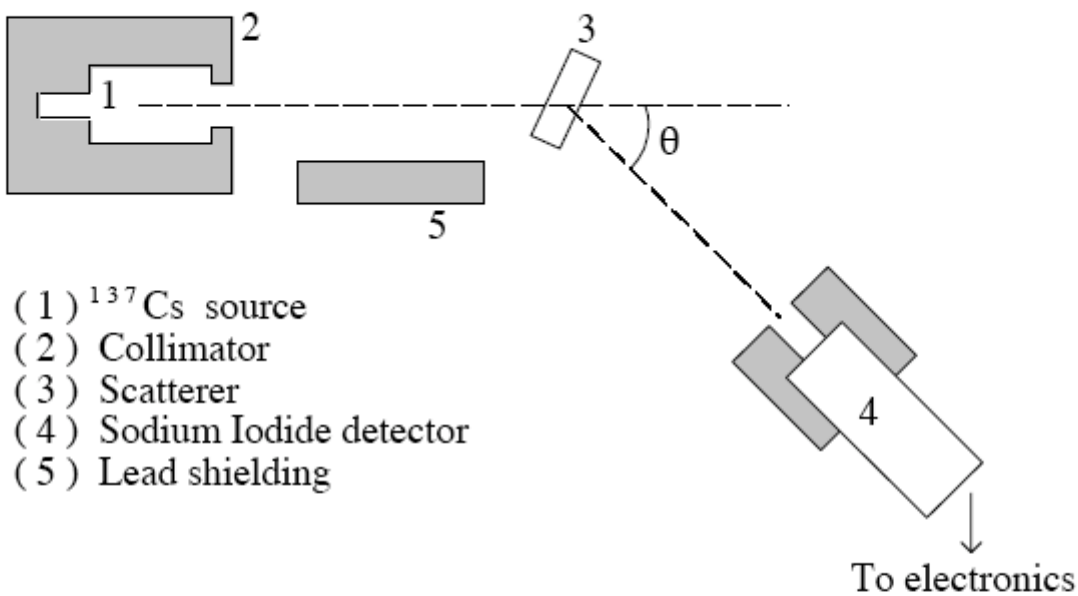

Fig. 2 Schematic diagram of apparatus for Compton scattering measurements

2. The axis of the collimator should be aligned to lie on the 0° scattering position of the counter. The γ-ray detector should be tested with the ^{137}Cs calibration source, with high voltage set to about 1100 V and the amplifier gain adjusted to give a pulse height of about 3 V for signals in the photoelectric peak of the spectrum. Check the calibration of the counter as it is set at different angles to be sure that it is not affected by stray magnetic fields.

3. Have the instructor install the source in the collimator. The source is relatively strong, and should be handled only by the instructor. Use the radiation survey meter to check radiation fields in the vicinity of the apparatus to ensure that there are no “leaks” in the shielding.

4. With no scatterer in place, observe the counting rate as a function of angle θ in the vicinity of 0°. This measurement will check that the collimator is properly aligned, and also provides a check of source strength. Because the source is strong, the counting rate will be high. This may result in *pulse pile-up* which can cause errors in the count rate and measured energy near 0°. Repeat the measurement with about 1 cm of lead in the beam as an absorber.

5. With the aluminum scatterer in place, measure the pulse height spectrum from the detector for a range of scattering angles between 0° and 120°. At each angle you should also measure the background spectrum with no scatterer in place, and subtract it from the spectrum of interest. You may be able to reduce the background by rearranging some of the lead shielding blocks.

6. From your data, compare the energy of the scattered γ-rays with the theoretical prediction.

7. Use your data to deduce the Compton scattering cross section per electron as a function of angle and compare with theoretical prediction.

8. Repeat 4, 5, and 6 at two or three angles for scatterers of lucite and copper.

References:

[1] Melissinos, Experiments in Modern Physics, p.252-265.

[2] Eisberg & Resnick, Quantum Physics, Sec. 2-4.

[3] UWO Radioisotope Safety Manual.

Keywords:

Compton scattering
cross section
Thompson cross section for Compton scattering